Mathematical models in neuroscience

Cellular Neurosci. Module 3
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References for Lecture 1: Sejnowski, Koch and Churchland *Science* Vol. 241 (1988), Marder *J. Neurophysiol*. 91 (2004), Mogilner, Wollman and Marshall, *Developmental Cell*, 11 (2006)

Why math?

- A “crutch for our intuition, to help us bridge the gap between what we can see and what we can think about” (intuition for systems that are “too complex”)
- Investigate experimentally challenging questions
- Integrate across scales to understand emergent phenomena (eg certain types of oscillations, consciousness)

Note: a computer model and a mathematical model is not necessarily synonymous

Goals and desirable outcomes of quantitative models

- Recreate the observed behavior (constraints on parameter values? Which parameters/variables matter most – what is a minimal model of behavior?)
- Quantitative hypothesis to be tested empirically
- Data interpretation
- Tying together phenomena and observations across different scales

Challenges in modeling

- Scale: often multiple scales interact and need to be considered (many ion channels producing an action potential; an action potential traveling down the axon) – question: what behavior/phenomenon are we interested in?
- Finding the appropriate methods (linked to scale)
- Finding appropriate questions: balance between sufficient quantitative data and room for different (quantitatively testable) hypotheses
- Cartoon vs photograph (what level of detail to include? Occam's Razor)

Different mathematical tools used in modeling

- Algebraic equations (describe functional relationships between unknowns)
- Ordinary differential equations (ODEs) rate of change (usually w/ respect to time)
- Partial differential equations (PDEs) rate of change in space and time
- Various stochastic models (Markov chains) probability associated with processes – stochastic models
- Graph theory methods

Typical course outline (Introduction to Computational Neurosci)

- Ion channel activation
- Action potential generation and Hodgkin-Huxley model
- Integrating inputs (synapses, dendrites, Hebb's Law, integrate and fire neurons, spike train analysis)
- Modeling tissue or an entire circuitry of the nervous system (modeling visual processing)

A dynamical systems perspective

- Differential equations
- Solutions to differential equations: a gold standard
- Equilibria
- Stability of equilibria
- Dynamical systems: understanding the “limiting behavior” of differential equations through studying the stability of equilibria and changes in the stability (bifurcations)