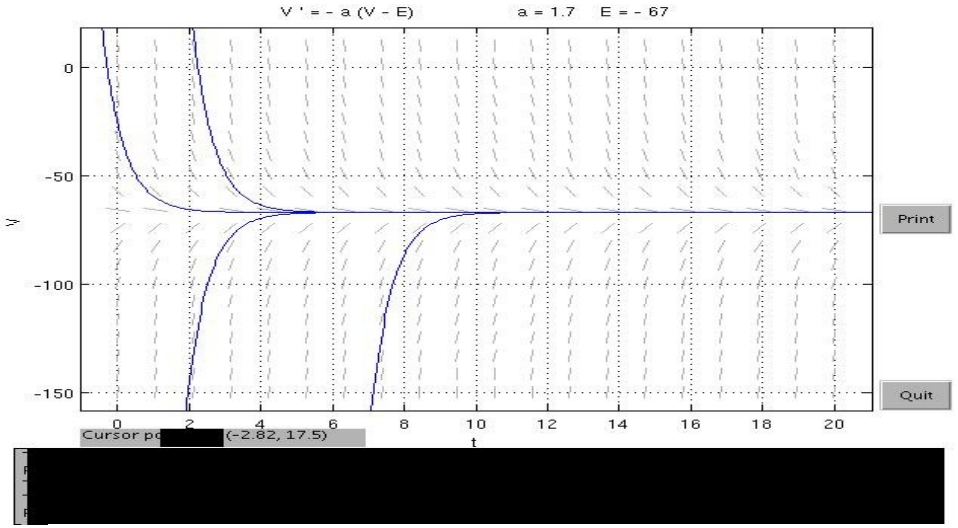
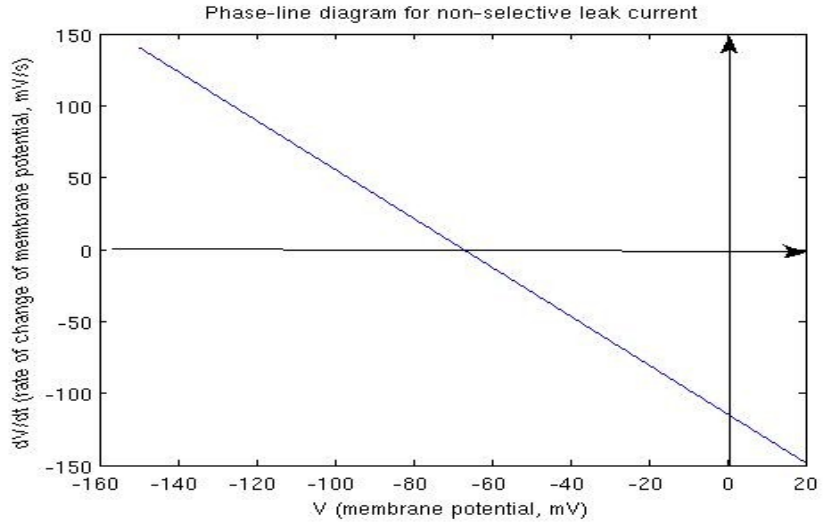
Phase lines and bifurcations

Lecture 3

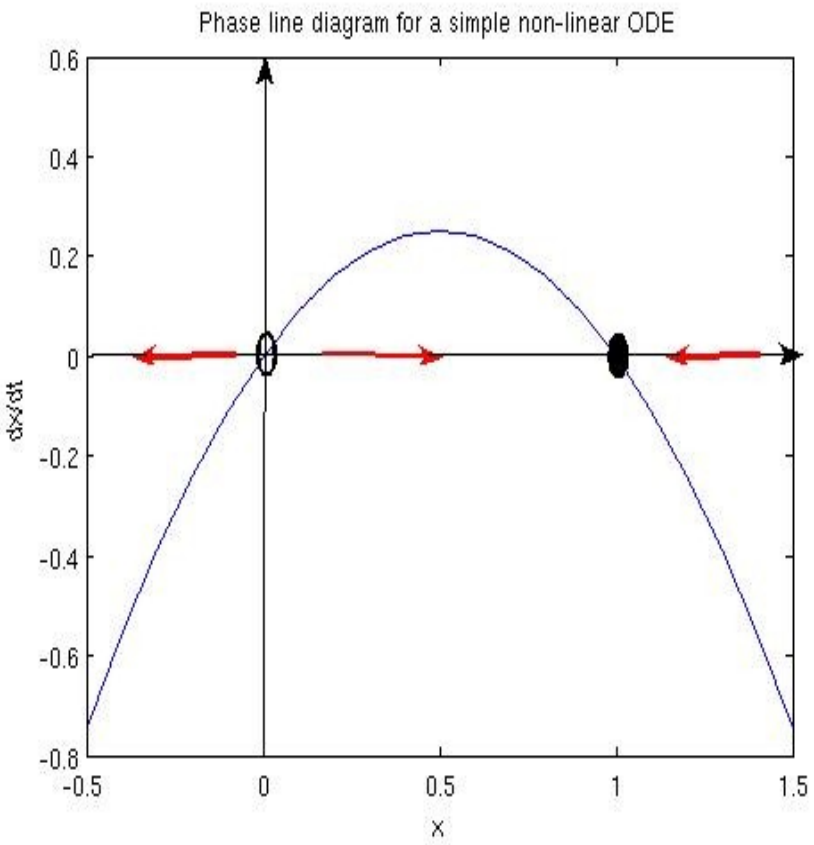
Direction fields and phase-line diagrams: a summary





- Direction field: gives the derivative at any (t,V)
- Phase-line diagram: plots derivative function, dV/dt versus V
- Representation of equilibria in figures

A quick example:

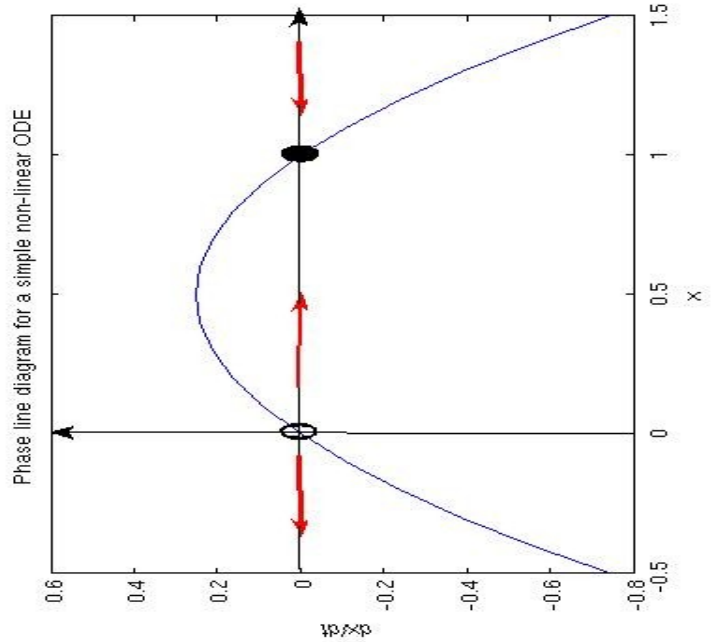


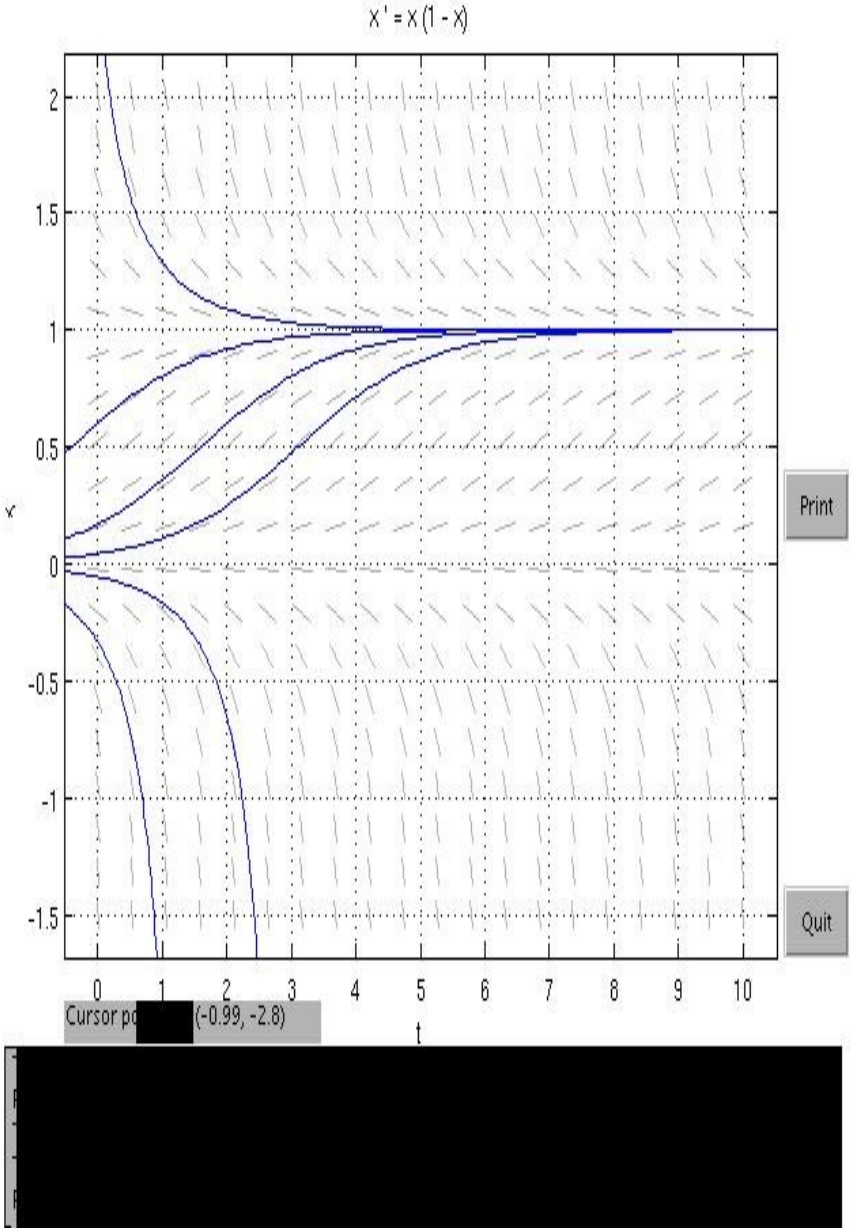
- Consider

  dx/dt = x(r-x)

  For now, let us work with r=1.

Phase-line and direction field





Bifurcations

- Definition: A bifurcation is a change in the number of equilibria or in their stability.
- Bifurcations occur when a parameter (called “bifurcation parameter”) is varied
- Bifurcations give rise to sudden changes in behavior of a system

Thresholds in development

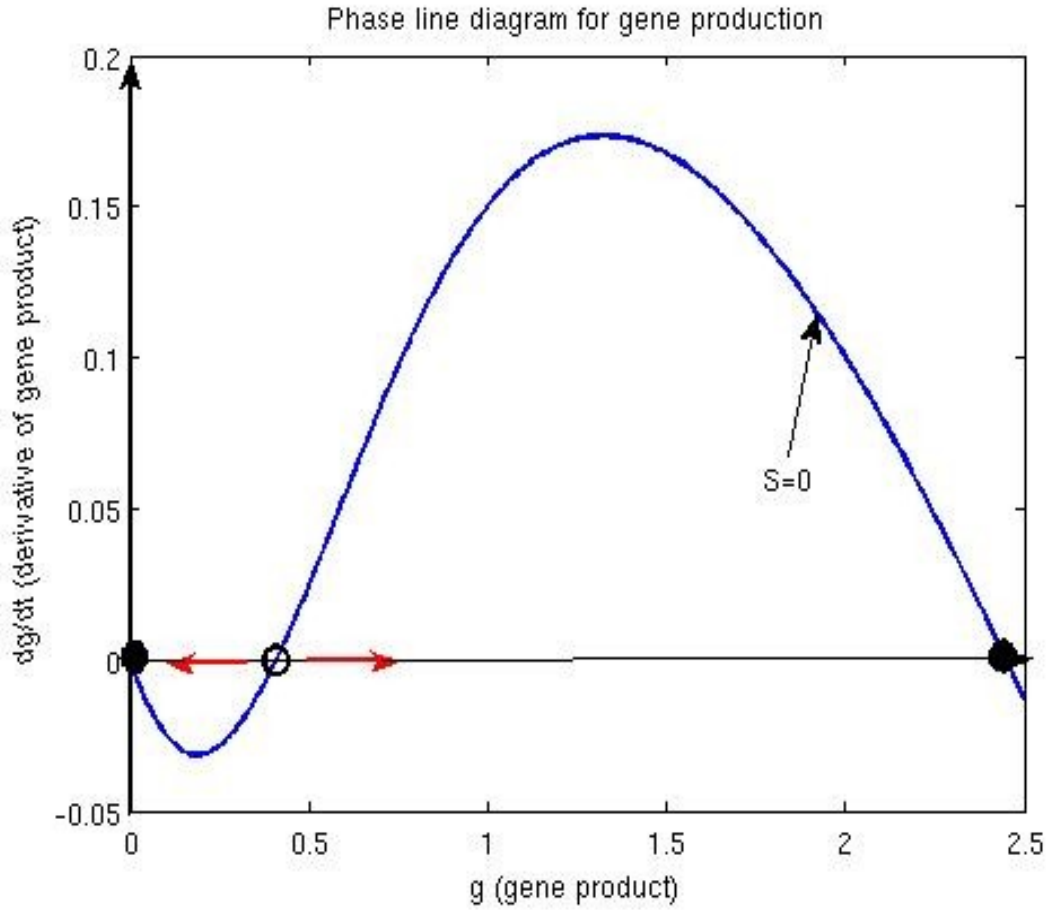
- Basic question: how can you turn a smooth gradient of external guidance cues into a sharp on-off switch
- Relevance to neuroscience: fate determination of various neural progenitor cells
- How good of a model would this be for neural development?

The model

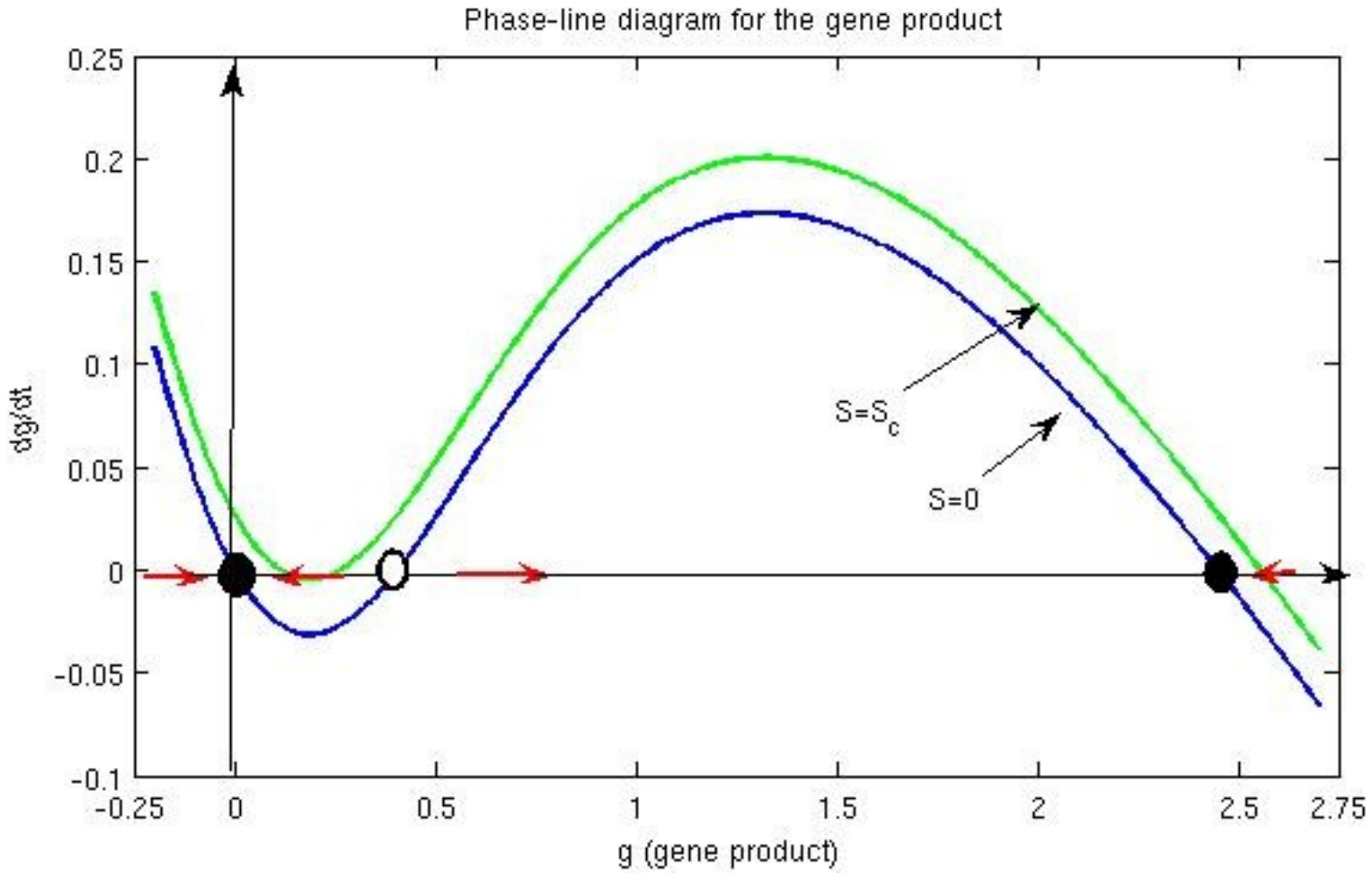
- Explanation of the terms of the model: Michaelis-Menten kinetics (autocatalytic production of g); linear decay; production based on external signal, S
- Start analysis by assuming that S=0

$$\frac{dg}{dt} = S + k_1 \frac{g^2}{k_2 + g^2} - k_3$$

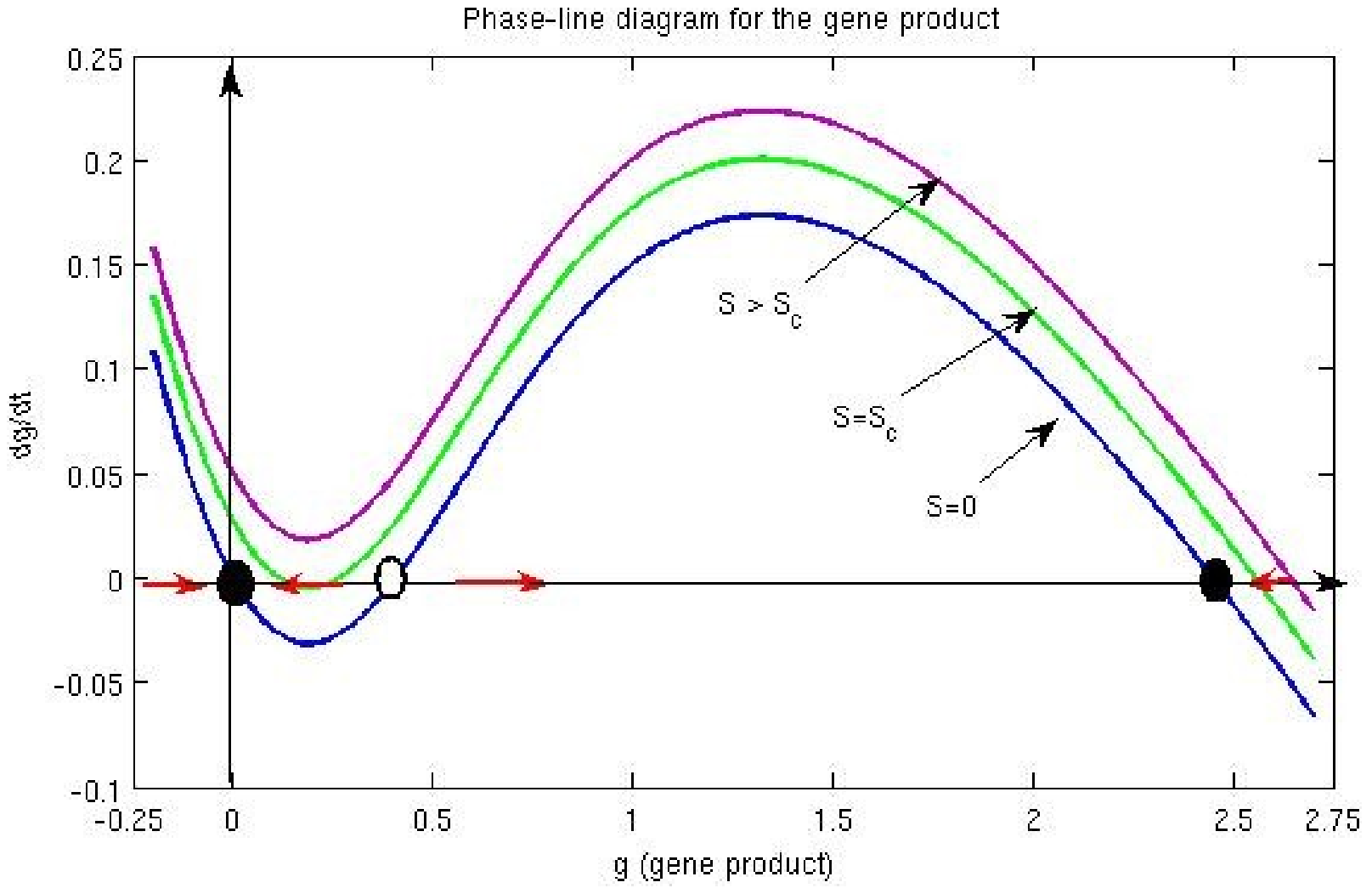
Phase line diagram for S=0



Increasing the signal-dependent production rate of the gene product



Increasing the signal-dependent production rate of the gene product



Bistability and hysteresis

- Now consider the the observed concentration of the gene product as the signal is decreased
- Hysteresis: memory in the system
- How does our system maintain a “memory”?
- The role of bifurcations in neuroscience: working toward the Hodgkin-Huxley model of action potential generation